MATH 590: QUIZ 7 SOLUTIONS

Name:

For the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, find:

1. The characteristic polynomial $p_A(x)$. (3 points)

Solution. $p_A(x) = \begin{vmatrix} x-1 & 0 & -1 \\ 0 & x-1 & 0 \\ -1 & 0 & x-1 \end{vmatrix}$. Expanding along the second row, we have

$$p_A(x) = (x-1) \cdot \begin{vmatrix} x-1 & -1 \\ -1 & x-1 \end{vmatrix} = (x-1) \cdot \{(x-1)^2 - 1\} = (x-1) \cdot (x^2 - 2x) = x(x-1)(x-2).$$

2. The eigenvalues of A. (3 points)

Solution. The eigenvalues of A are the roots of $p_A(x)$, which are $0, 1, 2$.

3. A basis for the eigenspace of each eigenvalue. (4 points)

Solution. E_0 is the nullspace of $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \xrightarrow{\text{EROs}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. A basis for the solution space of this last matrix is $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$, and hence is a basis for E_0.

E_1 is the nullspace of $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \xrightarrow{\text{EROs}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. A basis for the solution space of this last matrix is $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, and hence is a basis for E_1.

E_2 is the nullspace of $\begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \xrightarrow{\text{EROs}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. A basis for the solution space of this last matrix is $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, and hence is a basis for E_2.